Useful simplifications in the analysis of aircraft/aeroengine operation (page 25):

Chapter 3 in booklet.

(1) Analysis of aircraft/aeroengine operation is done in the aircraft frame-of-reference.

The engine is "stationary".

Inlet flow: V (flight velocity)   Exit flow: V_j (jet velocity)] i.e. relative to engine

(2) Core jet velocity = Bypass jet velocity = Jet velocity

$$V_{cj} = V_{bj} = V_j$$

– i.e. single, uniform jet velocity

(3) Usually the air mass flow rate ≈ 50 to 300 × fuel mass flow rate

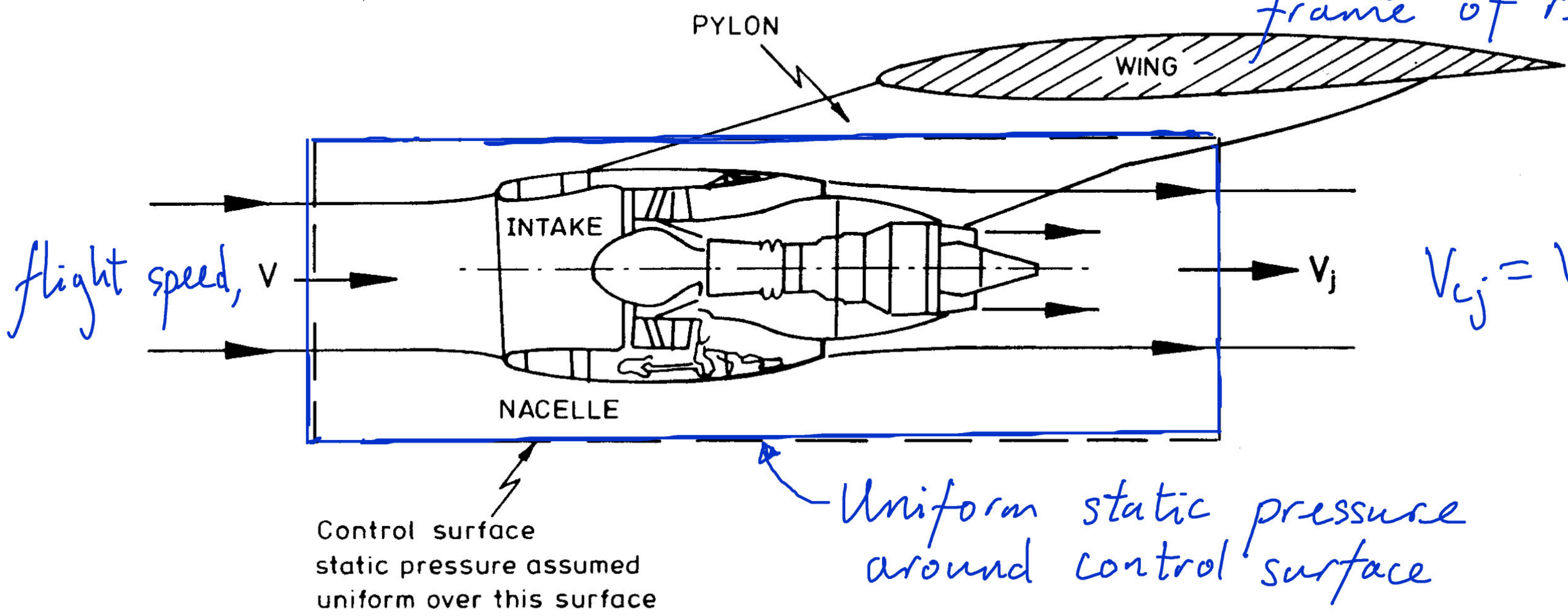
$$\dot{m}_a \approx 50 \text{ to } 300 \times \dot{m}_f$$

] very high air-to-fuel ratio

So may neglect $\dot{m}_f$ everywhere except for heat release (LCV = 43 MJ/kg).

Generation of thrust to propel the aircraft (page 25):

Engine (and aircraft) frame of reference



$V_{cj} = V_{bj} = V_j$

Uniform static pressure around control surface

	Inlet	Exit
Velocity	V	V_j
Momentum	$\dot{m}_a V$	$\dot{m}_a V_j$

} No pressure terms needed

Apply SFME:

Net thrust: $F_N = \dot{m}_a (V_j - V)$

V: Known (from $M = 0.85$ for $\left(\frac{ML}{D}\right)_{max}$)

F_N: Known (from thrust required by aircraft)

How to fix $\dot{m}_a$, V_j?

Relationship to thrust measured on the ground (where V = 0) (page 26):

When tested on the ground, there is no inlet momentum (ram drag) so the engine produces:

$$\text{Gross thrust: } F_G = \dot{m}_a V_j \quad \text{— (equation 3.2)}$$

This is the number quoted as the engine "thrust".

These two are related by: $F_N = \dot{m}_a (V_j - V) = \dot{m}_a V_j - \dot{m}_a V$

$$\text{Net thrust = Gross thrust – Ram drag}$$

$$F_N = F_G - \dot{m}_a V$$

⇒ Thrust ↓ as V↑ and as altitude ↑ (ρ↓)

The ram drag is the drag effect of the inlet air being "rammed" into the engine:

$$\text{Ram drag} = \dot{m}_a V$$

Need an efficiency measure to show how to "optimise" $\dot{m}_a$ and V_j for the engine.

Energy conversion process: “fuel” to “flight” (pages 27, 28, 29):

Energy "lost" in heating jet

Energy "lost" in excess jet K.E.

fuel heat release $\dot{m}_f LCV$ → power to jet ΔKE → power to aircraft $V F_N$

thermal efficiency (η_{th})

$$\eta_{th} = \frac{\Delta KE}{\dot{m}_f LCV}$$

propulsive efficiency (η_p)

$$\eta_p = \frac{V F_N}{\Delta KE}$$

overall efficiency (η_o)

$$\eta_o = \frac{V F_N}{\dot{m}_f LCV}$$

$\eta_{th} \Rightarrow$ how well the engine is designed

$\eta_p \Rightarrow$ how well the engine is matched to the aircraft.

fuel flow → (K.E.) power to jet

Thermal efficiency (page 28):

$$\eta_{th} = \frac{\Delta KE}{\text{fuel heat release}} = \frac{\Delta KE}{\dot{m}_f \, LCV} = \frac{\frac{1}{2}\dot{m}_a (V_j^2 - V^2)}{\dot{m}_f \, LCV}$$

← $\dot{m}_f$ neglected in ΔKE

LCV is the lower calorific value of the fuel, which is the energy released on complete combustion if the water in the products is not condensed but remains as vapour.

Later we will see that the thermal efficiency is closely related to the thermodynamic cyclic efficiency.

$\eta_{th} \approx \eta_{cycle}$

both measure thermodynamic performance of the engine.

Power (K.E.) to jet → Power (propulsion) to aircraft

Propulsive efficiency (page 27):

$$\eta_p = \frac{\text{power to aircraft}}{\text{power to jet}} = \frac{\text{power to aircraft}}{\Delta KE}$$

Power to aircraft = flight speed × net thrust:

$$= V \times F_N$$

$$= V\,\dot{m}_a(V_j - V)$$

$V \times F_N = V \times D$ = rate of doing work against drag.

Change in kinetic energy of airflow through the engine:

$$\Delta KE = \tfrac{1}{2}\dot{m}_a(V_j^2 - V^2)$$

Note, $(V_j^2 - V^2) = (V_j + V)(V_j - V)$

The propulsive efficiency:

$$\eta_p = \frac{V\,\dot{m}_a(V_j - V)}{\tfrac{1}{2}\dot{m}_a(V_j^2 - V^2)} = \frac{2V}{V_j + V}$$

Known as the *Froude* equation for propulsive efficiency.

If $V_j = V$, $\eta_p = 1$

but, $F_N = \dot{m}(V_j - V_0) = 0$

NO THRUST!

Overall efficiency (page 28):

fuel flow → useful power to aircraft

$$\eta_o = \frac{\text{power to aircraft}}{\text{fuel heat release}} = \frac{\text{power to aircraft}}{\Delta KE} \times \frac{\Delta KE}{\text{fuel heat release}} = \eta_p \times \eta_{th}$$

$\eta_o = \eta_p \times \eta_{th}$

This can be expanded into:

$$\eta_o = \frac{\text{Thrust} \times \text{speed}}{\dot{m}_f \, LCV} = \frac{\text{Thrust}}{\dot{m}_f} \times \frac{\text{speed}}{LCV} = \frac{1}{sfc} \times \frac{V}{LCV}$$

$sfc = \frac{\dot{m}_f}{F_N}$

$$\Rightarrow sfc = \frac{1}{\eta_o} \cdot \frac{V}{LCV} = \frac{1}{\eta_p \eta_{th}} \cdot \frac{V}{LCV}$$

Require low sfc at high flight speed (note - sfc varies with V)

What do these efficiencies tell us about aircraft/aeroengine design? (page 29):

Overall efficiency:

good match to aircraft/flight condition

good design of engine

$$\eta_o = \eta_p \times \eta_{th}$$

Require high propulsive efficiency and high thermal efficiency.

Propulsive efficiency:

$$\eta_p = \frac{2V}{V_j + V} = \frac{2V}{2V + (V_j - V)}$$

⇒ Need to match $V_j \rightarrow V$ so that $(V_j - V)$ is small

Require jet speed V_j to be only slightly higher then the flight speed V.

However: Thrust must be maintained,

needs to be large

needs to be small

Net thrust: $F_N = \dot{m}_a (V_j - V)$

so need a very large air mass flow through the engine. — Suggests a bypass engine!