

Not in the booklet!

Consequences of Breguet range equation (Extended material):

$$\text{Distance travelled} = \frac{V L/D}{g \text{ sfc}} \times \ln\left(\frac{w_{\text{start}}}{w_{\text{end}}}\right) \quad (\text{Equation 2.2, page 20})$$

Assumed constant

Cruise at 31000 ft:

$$T = 226.7 \text{ K} \quad (\text{from standard atmosphere}).$$

$$V = M\sqrt{\gamma RT} = 0.85\sqrt{1.4 \times 287 \times 226.7} = 256.5 \text{ m/s}$$

$$M = 0.8, C_L = 0.5$$

\Rightarrow cruise at 31000 ft

Specific fuel consumption:

$$\text{sfc} = 1.61 \times 10^{-5} \text{ kg s}^{-1} \text{ N}^{-1}$$

$$\text{sfc} = \frac{\dot{m}_f}{F_N} = \frac{\text{fuel flow}}{\text{net thrust}} \quad (\text{we shall see how to calculate this later})$$

Aircraft lift-to-drag ratio:

$$L/D = 20 \quad (\text{from design spec.})$$

Approximate weights (actually mass!) for New Large Aircraft (Extended material):

Based on Table 1.1

Empty weight (including fuel reserve of 10%): $w_e = 298.7 + 27.5 = 326.2$ tonne

fuel that is carried but not burned

Payload (passengers & cargo): $w_p = 58.8$ tonne

Fuel (90% of maximum, i.e. excluding reserve): $w_f = 250.6$ tonne

the fuel burned in flight

Combining these yield:

$$W_{\text{start}} = w_e + w_p + w_f = 635.6 \text{ tonne}$$

$$W_{\text{end}} = w_e + w_p = 385.0 \text{ tonne}$$

*≈ 40% of initial mass
is fuel burn
(c.f. ≈ 5% for a car)*

Note: no account of fuel used in climb/decent, hence figures are approximate.

significant for short flights

Estimated range for New Large Aircraft (Extended material):

$$\text{range} = \frac{V L/D}{g \text{ sfc}} \times \ln\left(\frac{w_{\text{start}}}{w_{\text{end}}}\right)$$

$$\text{range} = \frac{256.5 \times 20}{9.81 \times 1.61 \times 10^{-5}} \times \ln\frac{635.6}{385.0} = 16300 \text{ km}$$

1 nautical mile = 1 minute at Equator

$$\frac{16300 \text{ km}}{1.852 \text{ km/nm}} = 8800 \text{ nautical miles}$$

Converting to nautical miles (1 nautical mile = 1.852 km):

$$\text{range} = 8800 \text{ nautical miles (for 250.6 tonne of fuel)} > \frac{1}{3} \times \text{circumference of Earth!}$$

Given that the fuel burnt during climb has been ignored, this range is sufficient.

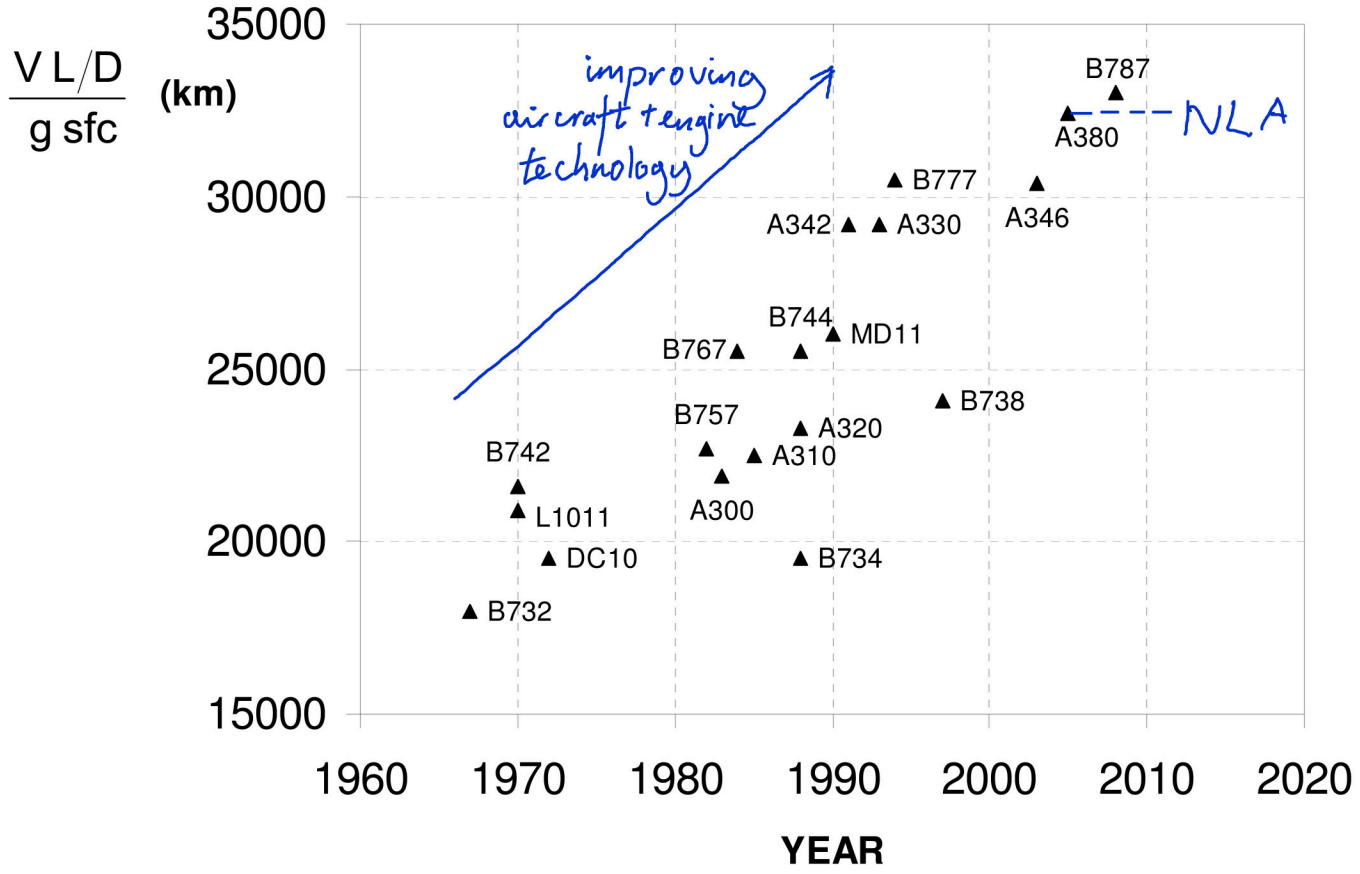
(This is basically Exercise 2.4, but with slightly different weights.)

Historical variation of the Breguet range parameter, $\frac{VL/D}{g \text{ sfc}}$ at cruise

For NLA,

$$\frac{VL/D}{g \cdot \text{sfc}} = \frac{256.5 \times 20}{9.81 \times 1.61 \times 10^{-5}}$$

$$= \underline{\underline{32480 \text{ km}}}$$



Fuel usage for flight with "stop-over" (Extended material):

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$$\frac{16300}{2} = 8150$$

How much fuel would be used for 8150 km flight (i.e. half of the above range)?

Assume w_e, w_p same $\Rightarrow w_{end}$ same, w_{start} changes

$$\frac{w_{start}}{w_{end}} = \exp\left\{\frac{s g sfc}{V L/D}\right\} = \exp\left\{\frac{8150 \times 10^3 \times 9.81 \times 1.61 \times 10^{-5}}{256.5 \times 20}\right\} = 1.285$$

Amount of fuel used:

$$w_f = w_{start} - w_{end} = \left(\frac{w_{start}}{w_{end}} - 1\right) w_{end} = (1.285 - 1) 385.0 = 109.7 \text{ tonne}$$

For two flights each of 8150 km,

total fuel used = 219.4 tonne

Single flight of 16300 km,

total fuel used = 250.6 tonne

*} i.e. saves 30 tonne
 $\approx 12\%$*

Considerations for flights with “stop-over” (Extended material):

(a) Less fuel is used:

Fuel is not used to carry fuel for later part of long flight.

This is less significant when fuel used in climb is included (short flights).

However, W_e can also be reduced as structure needs to support less weight.

(b) Allows better utilisation of aircraft:

Single aircraft can service more routes.

Have flexibility over which aircraft are used on each route.

i.e. reduces aircraft idle time.

(c) Disadvantages are:

Increased journey time.

Increased landing fees (noise, airport taxes).

Increased take-off and landing cycles (mechanical life).

Many components are limited to $\approx 10,000$ cycles

Fuel burn per kilometre ^{passenger + cargo weight} payload (Extended material):

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$$s = \frac{V L/D}{g \text{ sfc}} \times \ln\left(\frac{w_{\text{start}}}{w_{\text{end}}}\right) = \frac{V L/D}{g \text{ sfc}} \ln\left(\frac{w_e + w_p + w_f}{w_e + w_p}\right)$$

include 10% fuel reserves

So:
$$\frac{w_e + w_p + w_f}{w_e + w_p} = \exp\left\{\frac{s g \text{ sfc}}{V L/D}\right\}$$

$$1 + \frac{w_f}{w_e + w_p} = \exp\left\{\frac{s g \text{ sfc}}{V L/D}\right\}$$

$$w_f = (w_p + w_e) \left(\exp\left\{\frac{s g \text{ sfc}}{V L/D}\right\} - 1 \right)$$

Hence, fuel burn per kilometer payload:

$$\frac{w_f}{s w_p} = \frac{1}{s} \left(1 + \frac{w_e}{w_p} \right) \times \left(\exp\left\{\frac{s g \text{ sfc}}{V L/D}\right\} - 1 \right)$$

This is a measure of the efficiency of transport

Consequences of fuel burn per kilometre payload (Extended material):

$$\frac{w_f}{s w_p} = \frac{1}{s} \left(1 + \frac{w_e}{w_p} \right) \times \left(\exp \left\{ \frac{s g sfc}{V L/D} \right\} - 1 \right)$$

these need to be optimised.

(a) Minimize empty weight:

Lighter weight airframe and engine (25% of the A380 is composite)

Avoid “half-empty” planes (maximise aircraft load factors)

(b) Minimize sfc:

Best engine thermodynamics at cruise. (Good match of engine to aircraft.)
Next 2 lectures will investigate the factors that determine sfc.

(c) Maximize $V L/D$ (equivalent to $M L/D$):

Cruise at $M=0.85$ with $C_L = 0.5$

(i.e. need to optimise cruise condition with the wing technology)

Consequences of Breguet equation for emissions (extended material).

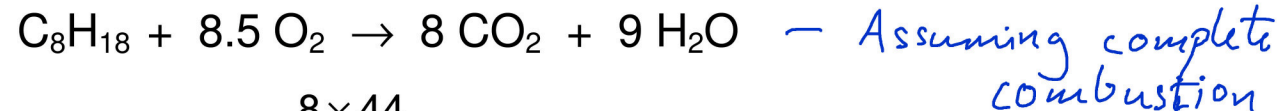
$$\text{Emission Index, EI} = \frac{\text{mass of pollutant}}{\text{mass of fuel burned}}$$

UNITS: g of pollutant / kg of fuel

For Carbon Dioxide (the primary greenhouse gas), the Emission Index only depends on the type of fuel. Assume an “average” chemical formula of C_8H_{18} for jet fuel.

*petroleum based,
kerosene*

The basic combustion reaction is therefore:



$$\text{Thus, EI}(\text{CO}_2) = \frac{8 \times 44}{8 \times 12 + 18} \approx 3100 \text{ g CO}_2 / \text{kg fuel}$$

The amount of CO_2 produced per passenger-km can now be calculated:

$$\frac{m_{\text{CO}_2}}{s N_{\text{pass}}} = \frac{m_f}{s m_p} \times \text{EI}(\text{CO}_2) \times m_{\text{pass}}$$

careful with units!

where m_{pass} = mass of 1 passenger with baggage ($\approx 100\text{kg}$).
↳ mass of payload

Use design range,⁴⁵
 $s = 8000 \text{ nm} = 14800 \text{ km}$

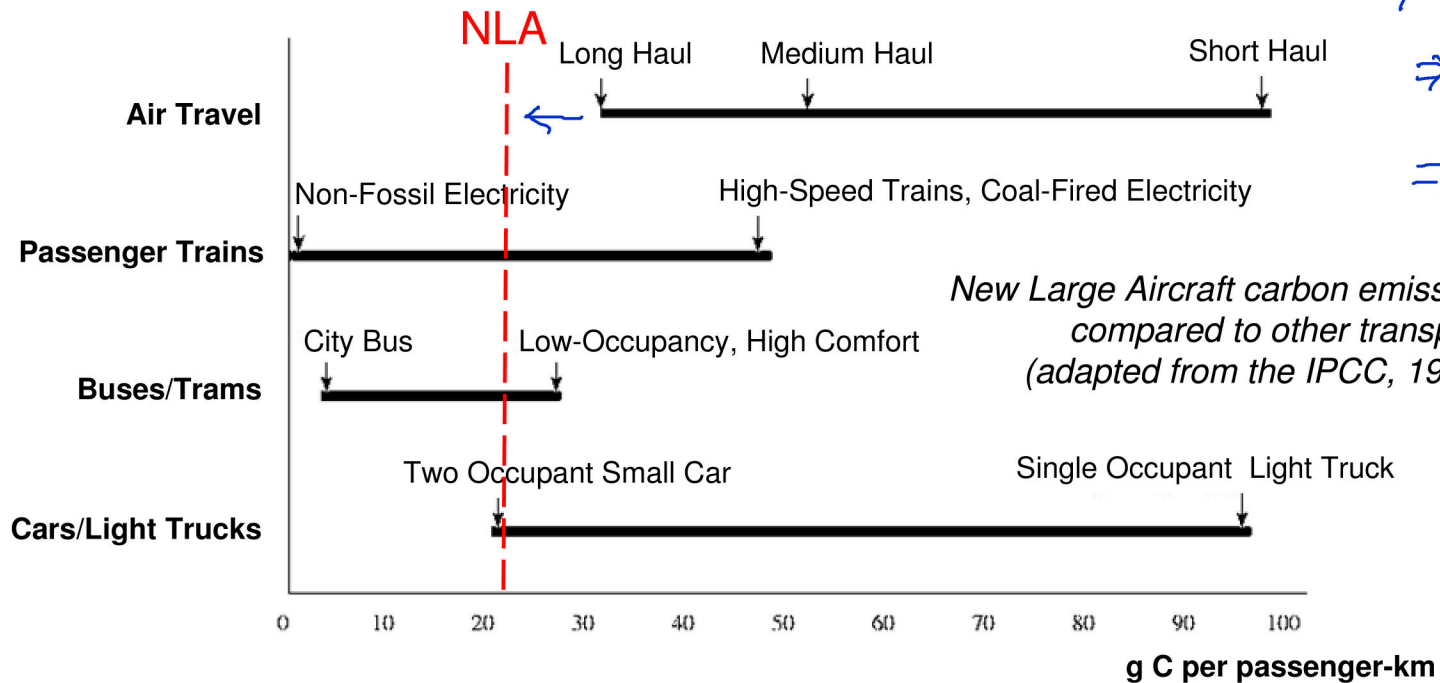
$$\text{For, our aircraft, } \frac{w_f}{s w_p} = \frac{1}{s} \left(1 + \frac{w_e}{w_p} \right) \times \left(\exp \left\{ \frac{s g \text{ sfc}}{V L/D} \right\} - 1 \right)$$

$$= \frac{1}{14800} \left(1 + \frac{326.2}{58.8} \right) \times \left(\exp \left\{ \frac{14800 \times 10^3 \times 9.81 \times 1.61 \times 10^{-5}}{256.5 \times 20} \right\} - 1 \right) = 0.255 \times 10^{-3} \text{ kg}/(\text{kg} \cdot \text{km})$$

Therefore, $\frac{m_{\text{CO}_2}}{s N_{\text{pass}}} = 0.255 \times 10^{-3} \times 3100 \times 100 = \underline{79 \text{ g CO}_2 \text{ per passenger-km}}$

km⁻¹ g/kg kg

79 g CO_2
 $\Rightarrow 79 \times \frac{12}{44} \text{ gC}$
 $= \underline{21.6 \text{ gC/pass-km}}$



New Large Aircraft carbon emission compared to other transport (adapted from the IPCC, 1999)

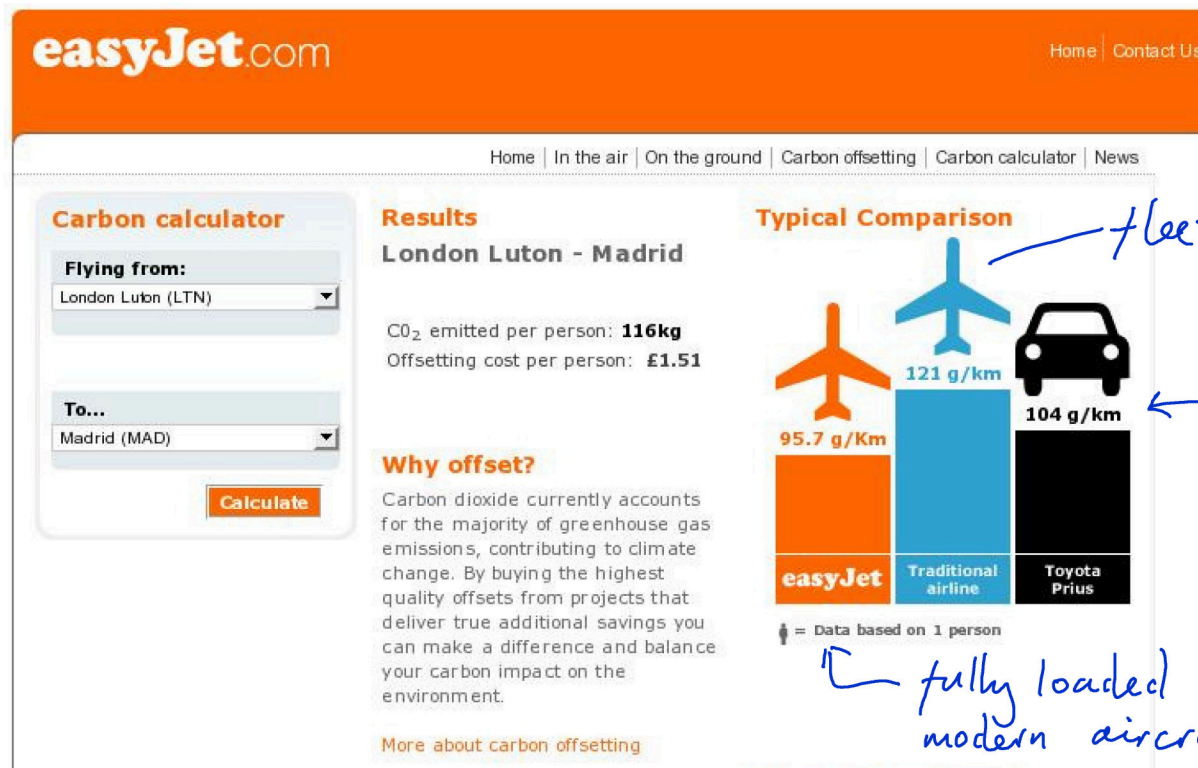
a bit out-of-date!

Emissions comparison (Extended material)

a bit optimistic! (higher payload, lower range?)

Airbus claims 75 g CO₂ per passenger-km for the fully-loaded A380, (www.airbus.com)

Results are sensitive to the actual passengers numbers and range flown. Low budget carriers claim “low emissions” due to high load factors and a modern fleet:



Comparison of CO₂ emissions from www.easyjet.co.uk