What determines the required wing area and the best cruising condition?

We shall show:

(a) Take-off conditions determine the (reference) wing area.

TAKE-OFF (This is the most demanding mechanical condition for the engine.)

- highest temperatures

(b) Maximum range determines cruising speed and altitude. max. range ≡ min. fuel burn

CRUISE (This is the most demanding thermodynamic condition for the engine.)

- highest efficiency

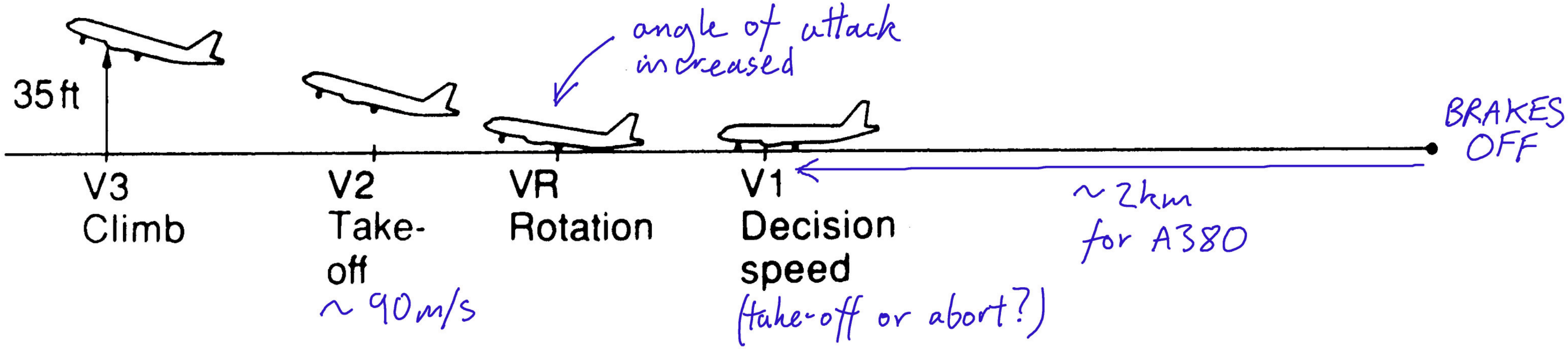
non-dimensionally

(c) Top-of-climb determines (maximum) engine thrust. → and sets the engine size.

TOP-OF-CLIMB (This is the most demanding aerodynamic condition for the engine.)

- low air density
- extra thrust needed to climb

Decision "points" (speeds) for take-off:



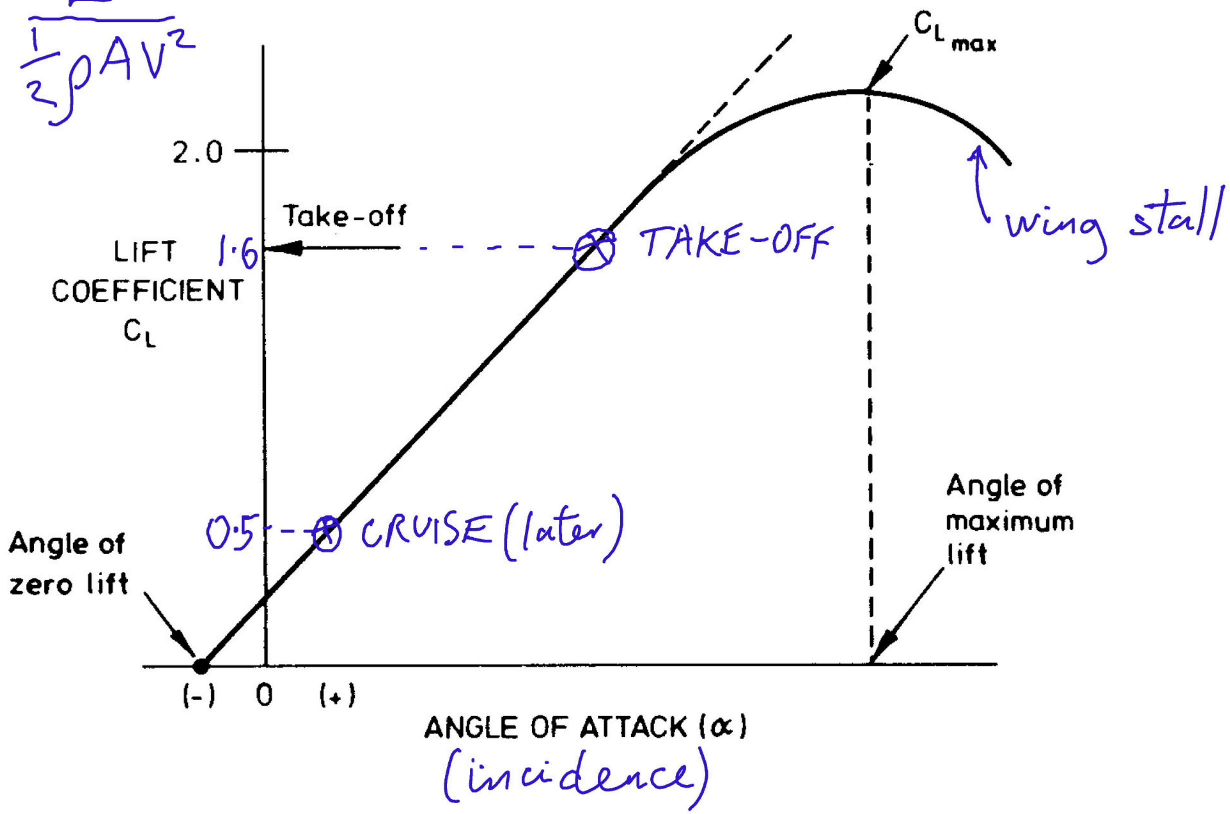
Critical points during take-off are set by speeds that are easily monitored by the pilot.

Runway length and tyre temperature limit take-off speed to 90 m/s (approx. 200 mph)

We shall see that take-off conditions essentially determine the wing area.

Typical lift-incidence curve for large subsonic civil transport at low-speed (page 16):

$C_L = \frac{L}{\frac{1}{2}\rho A V^2}$



Lift coefficient:

$$C_L = \frac{L}{\frac{1}{2}\rho A V^2} = 1\cdot 6 \text{ at take-off}$$

At sea level:

$$\rho = 1.225 \text{ kg/m}^3$$

At take-off:

$$V = 90 \text{ m/s}$$

At take-off, need a safety margin to avoid stalling the wings so choose: $C_L = 1.6$

Hence take-off condition determines required wing area A (page 16):

$$C_L = \frac{L}{\frac{1}{2}\rho A V^2}$$

Require that: Lift $\approx$ Weight $= 9.81 \times 635.6 \times 10^3$ N

$$A = \frac{L}{\frac{1}{2}\rho C_L V^2} \simeq 784\,m^2$$

i.e. very large!

Take-off at sea level (Exercise 2.1):

$C_L = 1.6$, $\rho = 1.225$ kg/m^3 and $V = 90$ m/s

Hence, wing area: $A \approx 784$ m^2

(This is the “reference” value for the wing area – true area is affected by slats & flaps.)

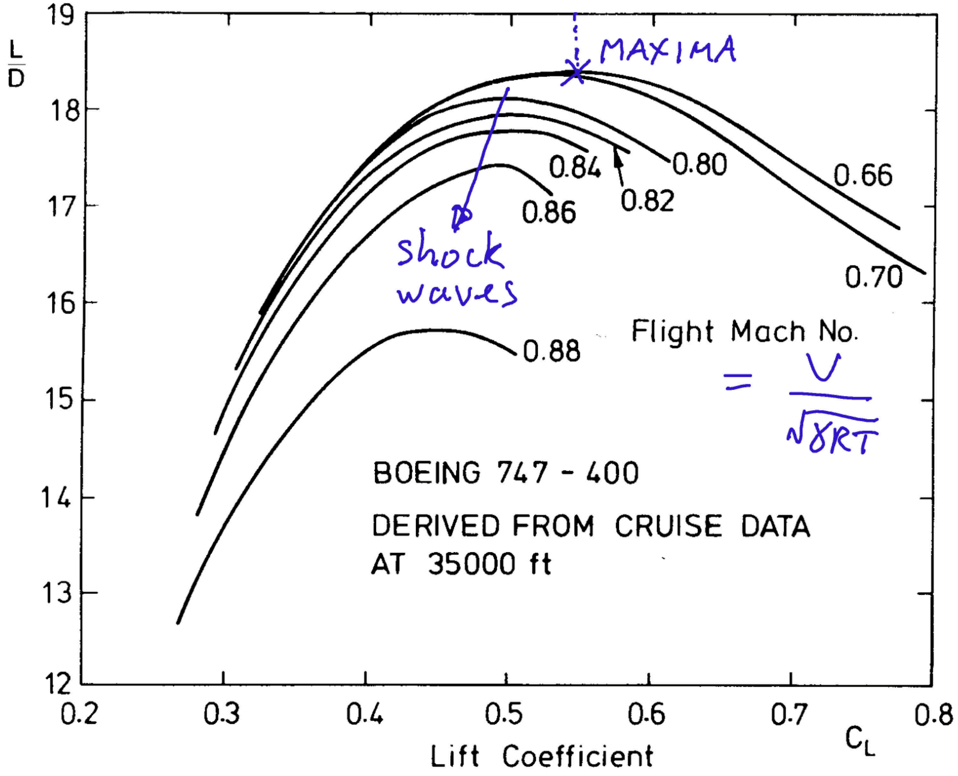
Note that at hot and/or high airports, $\rho \downarrow \Rightarrow$ harder to take-off.

How do we choose the cruising conditions for the aircraft? (page 17):

First idea would be to choose the cruise conditions to maximise the Lift/Drag ratio.

$\frac{C_L}{C_D} =$

i.e. $C_L \simeq 0.55$, $M \simeq 0.66$



This would yield the least drag associated with providing the necessary lift.

This would give us the smallest engine thrust required to power the aeroplane, but not necessarily the greatest range for the aircraft.

Engine (thrust) specific fuel consumption (sfc) (page 20):

sfc is a key quantity in determining the "quality" of an aeroengine.

$$\text{sfc} = \frac{\text{mass flow rate of fuel}}{\text{engine net thrust}}$$

– what you pay for

~ what the aircraft needs.

sfc is NOT non-dimensional

Correct units $\frac{\text{kg/s}}{\text{N}}$

Normal units $\frac{\text{kg/s}}{\text{kg}_f}$ or $\frac{\text{kg/hour}}{\text{kg}_f}$ where 1 kg of force = 9.81 N

The value of sfc is numerically equal in $\frac{\text{kg/s}}{\text{kg}_f}$ or $\frac{\text{lb/s}}{\text{lb}_f}$

The system based on pounds (lb) is the usual industry standard.

How to maximise the range of the aircraft (page 20):

As fuel is burnt to provide thrust the aircraft becomes lighter and so less lift is required. Thus less drag is produced so less thrust is required so fuel is burnt more slowly.

$$\mathrm{sfc} = \frac{\text{mass flow rate of fuel}}{\text{engine net thrust}} = \frac{\dot{m}_f}{F_N}$$

rate of change of aircraft weight $= \dfrac{dw}{dt} = -g\dot{m}_f = -g\,\mathrm{sfc}\,F_N = -g\,\mathrm{sfc}\times \mathrm{drag} = -g\,\mathrm{sfc}\times\dfrac{\mathrm{lift}}{L/D} = -\dfrac{g\,\mathrm{sfc}}{L/D}\times w$

$$\frac{dw}{w} = -\frac{g\,\mathrm{sfc}}{L/D}\times dt = -\frac{g\,\mathrm{sfc}}{L/D}\times\frac{ds}{V}$$

Integrating,

$$[\ln w]_{\mathrm{start}}^{\mathrm{end}} = -\frac{g\,\mathrm{sfc}}{V\,L/D}\times[s]$$

Obtain the Breguet Range Equation:

$W_{start} = 640$ tonnes
$W_{end} = 360$ tonnes

$$\underset{(S)}{\text{Distance travelled}} = \frac{V\,L/D}{\underbrace{g\,\text{sfc}}_{\text{assumed constant}}} \times \ln\overbrace{\left(\frac{w_{start}}{w_{end}}\right)}^{\sim 2} \qquad \text{(Equation 2.2, page 20)}$$

Hence to maximise the distance travelled (range) the quantity V L/D is important.

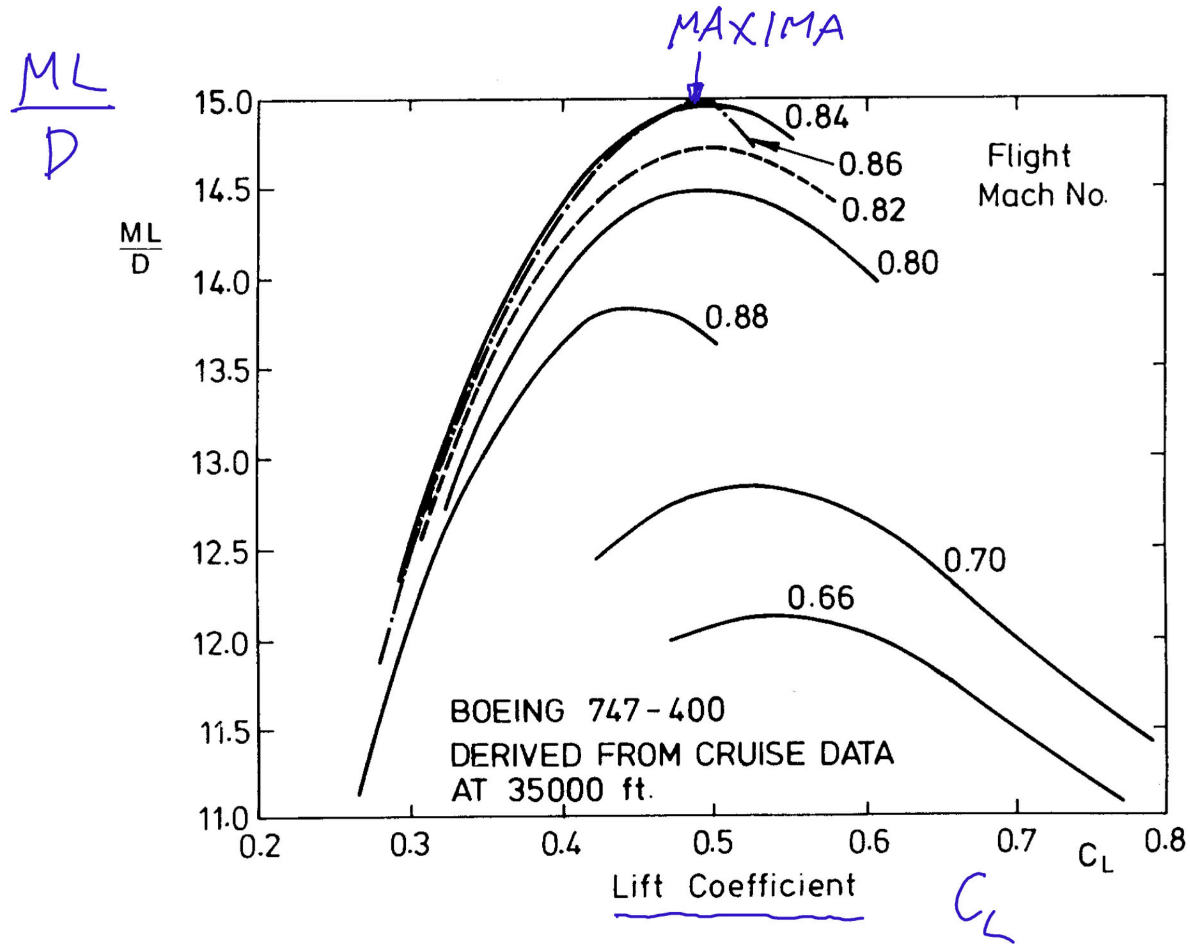
During cruise the ambient temperature varies little so the speed of sound is (almost) constant. Hence the maximum range is obtained by considering:

$$M\,L/D = \text{Mach} \times \text{Lift} / \text{Drag}$$

i.e. What is the flight condition that makes $\left(M \frac{L}{D}\right)$ maximum?

Mach number: $M = V/(\text{speed of sound}) = V/\sqrt{\gamma RT}$

Maximum range determines the cruising condition (page 19):



For cruise maximise:

M L/D

Hence, at cruise:

$$C_L = 0.5$$

and:

$$\text{Mach} = 0.85$$

$\left\} \text{for } \left(\frac{ML}{D}\right)_{max}\right.$

For a Boeing 747-400 this would give: L/D ≈ 17.5

New Large Aircraft: L/D = 20 $\Rightarrow \frac{ML}{D} \approx 17$

better wing technology

Lift coefficient at cruise determines why all large civil aircraft fly so high (page 15):

$M = 0.85$, $C_L = 0.5$ for $\left(\frac{ML}{D}\right)_{max}$

$$C_L = \frac{L}{\frac{1}{2}\rho A V^2}$$

Require that: Lift = Weight = $9.81 \times 635.6 \times 10^3$ N

$$V = \sqrt{\frac{\text{Weight}}{\frac{1}{2}\rho A C_L}}$$

Cruise at sea level:

$C_L = 0.5$, $\rho = 1.225$ kg/m^3 and $A = 784$ m^2

Hence: $V = 161$ m/s (Mach ≈ 0.47) - TOO SLOW

Cruise at 31000 ft:

(See Exercise 2.2)

$C_L = 0.5$, $\rho = 0.442$ kg/m^3 and $A = 784$ m^2

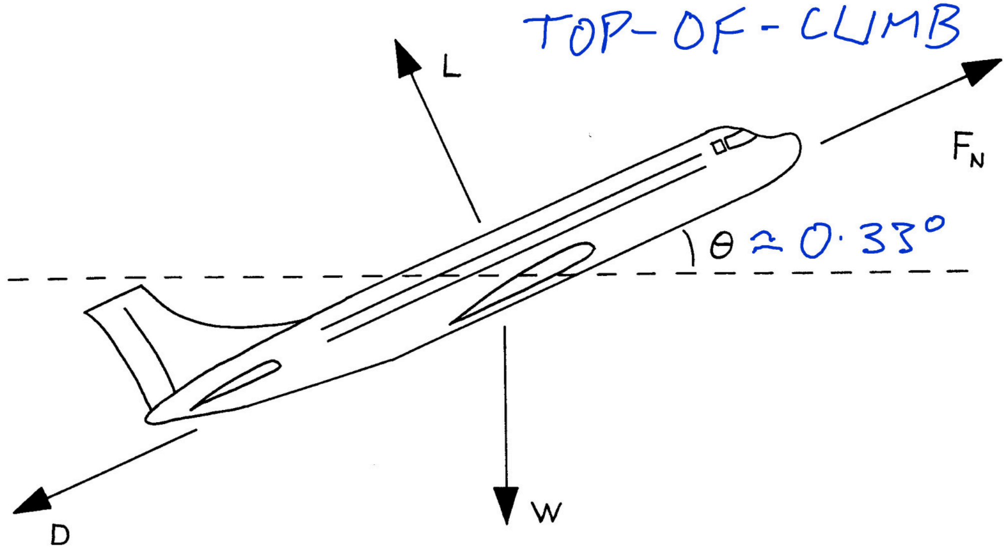
Hence: $V = 268$ m/s (Mach ≈ 0.89) - ABOUT RIGHT

$\Rightarrow$ cruise at 31000 ft

Whittle was right: high and fast!

So how much thrust must the engines provide? (page 22):

Most demanding aerodynamic condition for the engine is at top-of-climb where:



$$F_N / W \approx \underbrace{\frac{1}{L/D}}_{0.05} + \underbrace{\sin\theta}_{0.0058}$$

Thrust at top-of-climb determines engine size (see page 21)

Assuming that cruise starts at 31000 ft, M=0.85 (Exercise 2.3).

⇒ Top-of-climb is also at 31000 ft, M = 0.85

Once the fuel burn during climb has been estimated, the aircraft weight at the start of cruise is known and so the engine thrust at top-of-climb can be determined.

As more fuel is burnt, the aircraft climbs higher to maintain optimal value of C_L = 0.5

$$C_L = \frac{L}{\frac{1}{2}\rho A V^2}$$

See exercise 2.5.