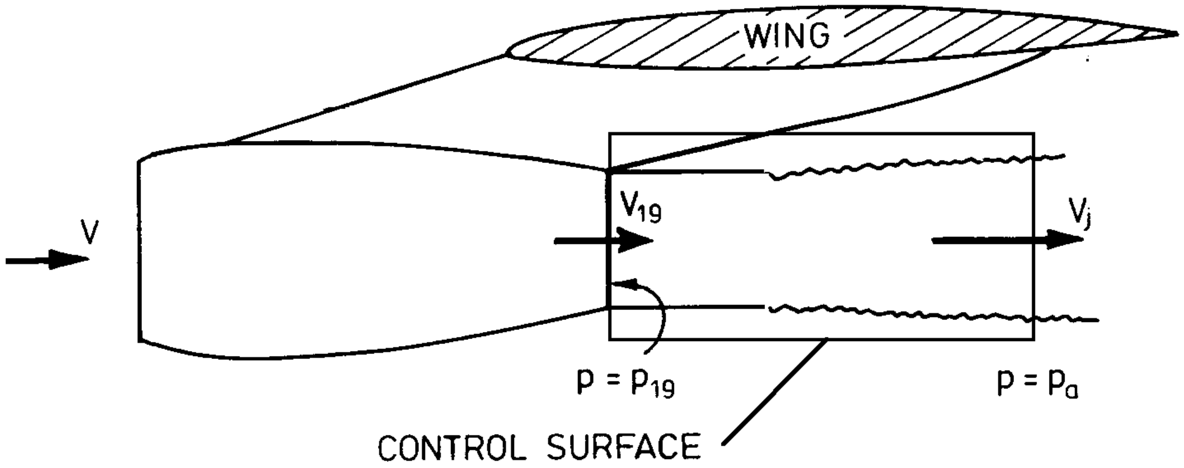
Civil aeroengines for subsonic cruise have convergent nozzles (page 83):

Choked convergent nozzle must be sonic at the exit A_N.

Consequently, the pressure (p_{19}) at the nozzle exit will be above the surrounding ambient pressure (p_a).



(based on Fig. 8.1)

There is no net force on the control surface so:

$$\text{Gross thrust} = F_G = \dot{m}_a V_j = \dot{m}_a V_{19} + (p_{19} - p_a) \times A_N \qquad \text{(Eqn 8.2)}$$

$$= [\, \dot{m}_a V_{19} + p_{19} \times A_N \,] - p_a \times A_N$$

Hence the gross thrust depends on the ambient pressure.

Earlier assumption for the calculation of thrust (page 83):

In earlier calculations the jet velocity was estimated by isentropically expanding the turbine exit flow to the ambient static pressure. Technically this is **incorrect** because downstream of a convergent nozzle there would be unconstrained expansion (shock-diamonds) which would be irreversible.

For most subsonic civil aircraft this approximation only introduces a small error. The correct method is to expand the turbine exit flow to sonic conditions (M=1) and then to calculate the gross thrust from the nozzle exit plane momentum and pressure contribution above ambient.

"Correct" gross thrust calculation for choked nozzle (Ex. 8.2):

Convergent bypass nozzle must have Mach = 1.0 at exit plane (location 19):

$$T_{19} = T_{013} \,/\, \left(1 + \tfrac{\gamma - 1}{2} M^2\right) = 312.8 \,/\, 1.2 = 260.7 \text{ K}$$

$$p_{19} = p_{013} \left(\frac{T_{19}}{T_{013}}\right)^{\gamma/(\gamma-1)} = 81.7 \times 10^3 \times \left(\frac{260.7}{312.8}\right)^{3.5} = 43.2 \text{ kPa}$$

$$V_{19} = 1.0 \times \sqrt{\gamma R T_{19}} = 323.6 \text{ m/s}$$

From earlier (A_{Nb} = 2.36 m^2, $\dot{m}_b$ = 440.5 kg/s and p_a = 28.7 kPa):

$$\text{Gross thrust} = F_G = \dot{m}_b V_{19} + (p_{19} - p_a) \times A_{Nb} = 176.8 \text{ kN}$$

If we make the "reasonable" assumption of isentropic flow (V_{jb} = 403 m/s):

$$\text{Gross thrust} = F_G = \dot{m}_b V_{jb} = 440.5 \times 403 = 177.5 \text{ kN}$$

Dynamic Scaling and Dimensional Analysis (page 81):

Now that the aeroengine is designed, it would be possible to calculate its performance, and hence the net thrust, for any aircraft operating conditions (altitude & speed).

However:

The engine almost always operates at the same temperature ratio ($T_{04}/T_{02} \approx 5.5$).

The engine internal performance is independent of the ambient pressure.

It is therefore possible to form non-dimensional groups which can be used to "dynamically scale" the basic cruising performance to many other conditions.

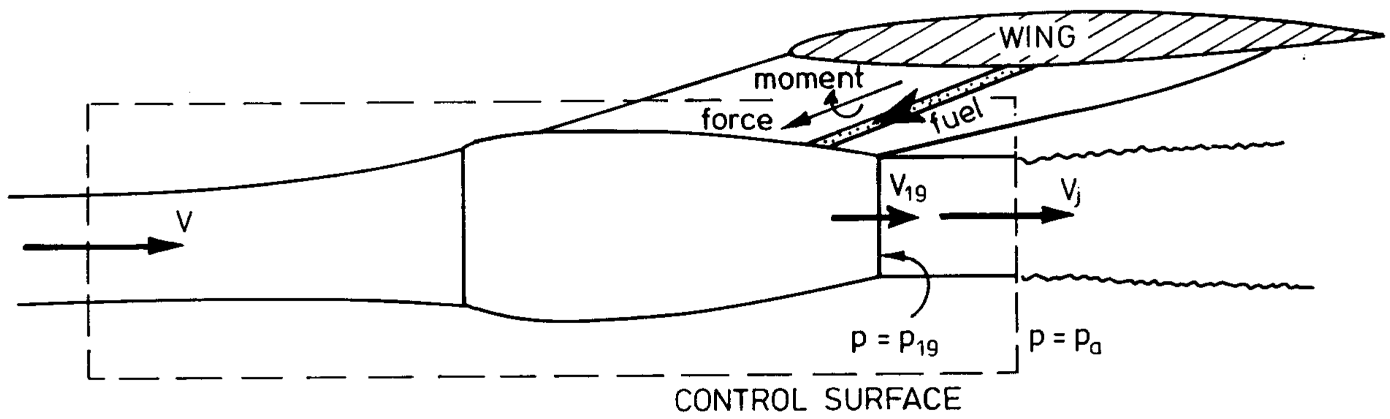
To do this we need to use physical understanding to decide how to form useful non-dimensional groups. (The maths. is easy. Understanding is **hard!**)

Engine internal performance (internal pressure & temperature) depends on (page 81):

The inlet stagnation pressure (p_{02}).

The inlet stagnation temperature (T_{02})

The fuel flow rate ($\dot{m}_f$)



(Fig. 8.1)

As the exit nozzle is choked, no information can pass upstream, so the engine internal performance does not depend on the exit static (ambient) pressure.

(The gross thrust, however, does depend on the ambient static pressure!!)

Engine performance depends on (page 80):

The inlet stagnation pressure (p_{02}).

The inlet stagnation temperature (T_{02}).

The fuel flow rate ($\dot{m}_f$). --- This is the only control the pilot has!

It also depends on:

The working fluid (air - γ, R fixed).

The size of the engine (D - fan diameter fixed).

The fuel type (LCV - fixed).

As these later variables cannot be changed, they will not be explicitly included in the function relationships. Only include them to make non-dimensional groups.

We will use "physical" insight to produce "useful" non-dimensional groups (hard bit!).

$\dot{m}_a$ = function ($\dot{m}_f$, p_{02} , T_{02}) (page 81)

If we think back to compressible flow and the non-dimensional mass flow, then:

$$\frac{\dot{m}_a \sqrt{C_p T_{02}}}{D^2 p_{02}}$$

Is a suitable non-dimensional air mass flow.

(Essentially this is related to the Mach number at any location within the engine.)

The fuel mass flow rate could be made non-dimensional in a similar manner. However, the fuel flow rate is a rate of heat input (power) and so ought to be thought of as a rate of working (ie force × velocity = $D^2 p_{02} \times \sqrt{\gamma R T_{02}}$). Thus:

$$\frac{\dot{m}_f \, LCV}{D^2 p_{02} \sqrt{C_p T_{02}}}$$

Is a suitable non-dimensional fuel flow rate.

Thus, using physical insight, air mass flow has the following dependence (page 82):

$$\frac{\dot{m}_a \sqrt{C_p T_{02}}}{D^2 p_{02}} = \text{function}\left(\frac{\dot{m}_f \, LCV}{D^2 p_{02} \sqrt{C_p T_{02}}}\right)$$

Turbine inlet temperature $T_{04} = \text{function}\,(\dot{m}_f\ , p_{02}\ , T_{02}\ )$ can be re-arranged as:

$$\frac{T_{04}}{T_{02}} = \text{function}\left(\frac{\dot{m}_f \, LCV}{D^2 p_{02} \sqrt{C_p T_{02}}}\right)$$

The engine rotational speed of any chosen shaft also has a similar dependence:

$$N = \text{function}\,(\dot{m}_f\ , p_{02}\ , T_{02}\ )\ ,$$

A suitable non-dimensional form is:

$$\frac{ND}{\sqrt{\gamma R T_{02}}} = \text{function}\left(\frac{\dot{m}_f \, LCV}{D^2 p_{02} \sqrt{C_p T_{02}}}\right)$$

The above relationships can be combined to give (page 82):

$$\frac{\dot{m}_a\sqrt{C_p T_{02}}}{D^2 p_{02}} = \text{function}\left(\frac{\dot{m}_f\, LCV}{D^2 p_{02}\sqrt{C_p T_{02}}}\right)$$

or:

$$\frac{\dot{m}_a\sqrt{C_p T_{02}}}{D^2 p_{02}} = \text{function}\left(\frac{T_{04}}{T_{02}}\right)$$

or:

$$\frac{\dot{m}_a\sqrt{C_p T_{02}}}{D^2 p_{02}} = \text{function}\left(\frac{ND}{\sqrt{\gamma R T_{02}}}\right)$$

Depending upon which variables are “available”, any of the above can be used.

Could use any reference area:

D^2 or A_N or A_{Nb} etc.

What is the take-off mass flow rate for conditions (T_{04}/T_{02}) similar to cruise (page 88):

At cruise (31000 ft, M = 0.85, $T_{04}/T_{02} = 5.6$):

$T_{02} = 259.5$ K, $p_{02} = 46.0$ kPa, $\dot{m}_a = 514$ kg/s, $A_N = 3.14$ m^2

$$\frac{\dot{m}_a \sqrt{C_p T_{02}}}{p_{02} \times A_N} = \frac{514 \times \sqrt{1005 \times 259.5}}{46.0 \times 10^3 \times 3.14} = 1.82 \text{ (no units!)}$$

At take-off (sea level, 90 m/s, $T_{04}/T_{02} = 5.6$):

$T_a = 288$ K, $p_a = 101.3$ kPa

$T_{02} = 292.0$ K, $p_{02} = 106.3$ kPa

$$\dot{m}_a = \frac{1.82 \times p_{02} \times A_N}{\sqrt{C_p T_{02}}} = \frac{1.82 \times 106.3 \times 10^3 \times 3.14}{\sqrt{1005 \times 292}} = 1121.4 \text{ kg/s}$$