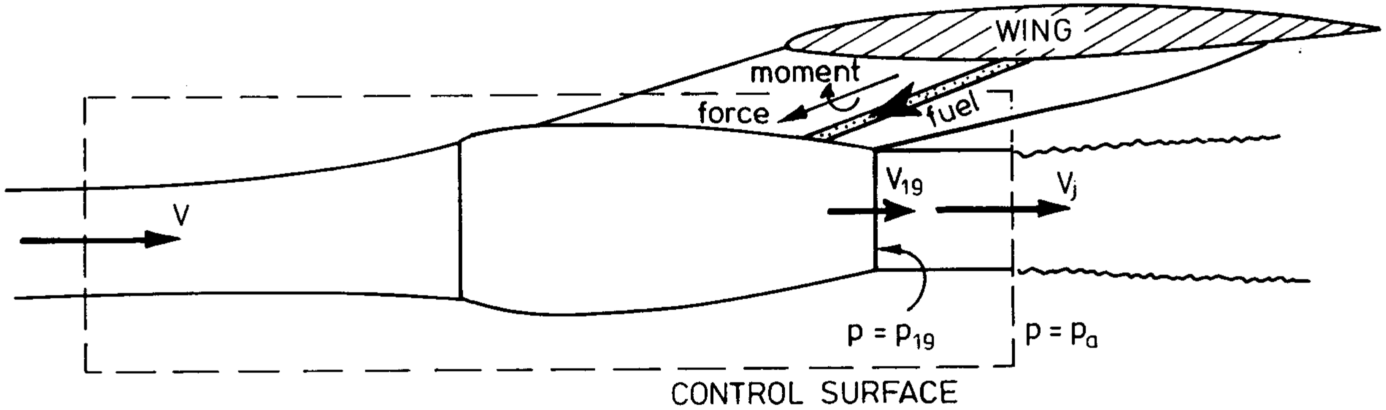
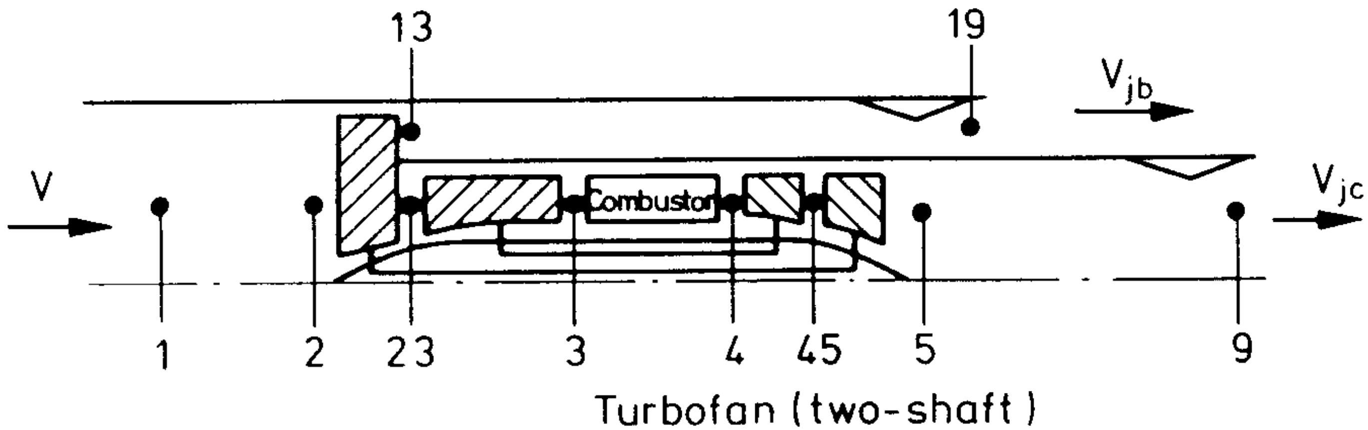


(fig 8.1, page 81)



(fig 7.1, page 70)

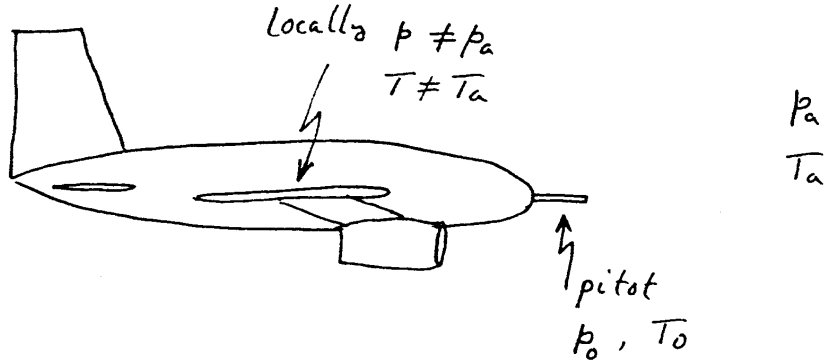
What are the engine inlet conditions T_{02} and p_{02} for flight conditions?

What are the conditions at bypass (and core) nozzle exit?

Engine inlet conditions are stagnation conditions (page 63):

Ambient (barometric) pressure and ambient temperature are **static** quantities.
Stagnation pressure and stagnation temperature depend on the frame of reference.

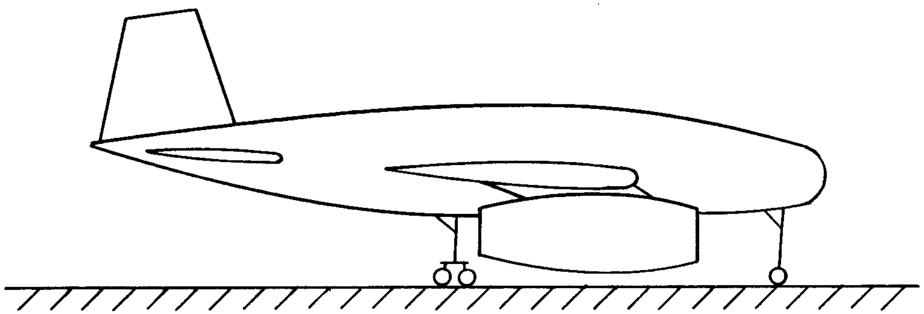
An observer travelling at velocity V, through an atmosphere with static temperature T_a and static pressure p_a would perceive a stagnation temperature T_{0a} and stagnation pressure p_{0a}.



Aeroengine inlet conditions are the perceived stagnation quantities T_{0a} and p_{0a}.

$$\frac{T_{0a}}{T_a} = 1 + \frac{\gamma - 1}{2} M^2 \qquad \text{and} \qquad \frac{p_{0a}}{p_a} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

Engine inlet conditions when on the ground (page 63):



At sea level:

$T_a = 288$ K

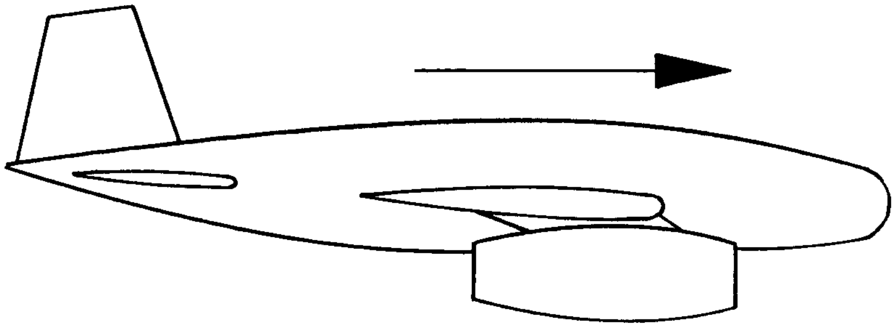
$p_a = 101$ kPa

On the ground, with zero forward speed: V = 0, (M = 0)

$$T_0 = T_a\left(1+\frac{V^2}{2C_pT_a}\right) = T_a\left(1+\frac{\gamma-1}{2}M^2\right) = 288\text{ K}$$

$$p_0 = p_a\left(\frac{T_0}{T_a}\right)^{\gamma/(\gamma-1)} = p_a\left(1+\frac{V^2}{2C_pT_a}\right)^{\gamma/(\gamma-1)} = p_a\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)} = 101\text{ kPa}$$

Engine inlet conditions when flying at 31000 ft (page 63 Ex 6.2):



At 31000 ft:

T_a = 227 K

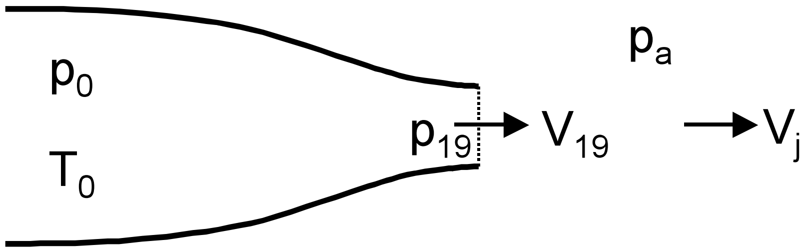
p_a = 28.7 kPa

At 31000 ft, with forward speed: V = 256 m/s, (M = 0.85)

$$T_0 = T_a\left(1+\frac{V^2}{2C_pT_a}\right) = T_a\left(1+\frac{\gamma-1}{2}M^2\right) = 259\text{ K}$$

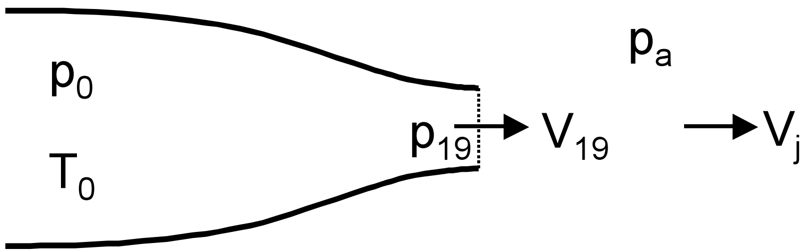
$$p_0 = p_a\left(\frac{T_0}{T_a}\right)^{\gamma/(\gamma-1)} = p_a\left(1+\frac{V^2}{2C_pT_a}\right)^{\gamma/(\gamma-1)} = p_a\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)} = 28.7\times1.60 = 45.8\text{ kPa}$$

Engine (bypass and core nozzle) exit condition (page 68):



Unchoked:

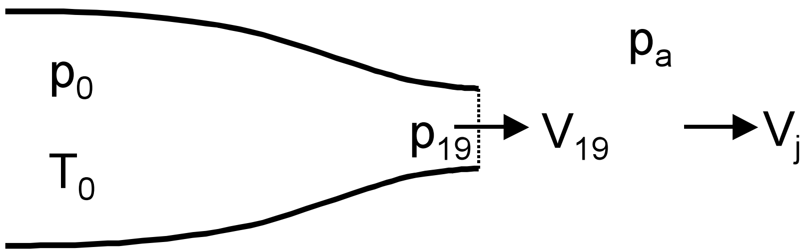
$M_{19} < 1.0$ and $p_{19} = p_a$



Just choked:

$M_{19} = 1.0$ and $p_{19} = p_a$

Maximum mass flow

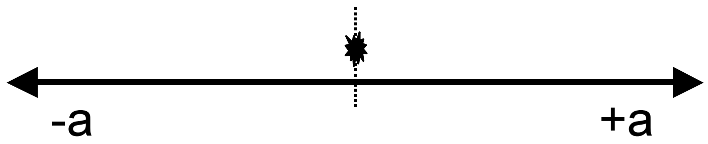


Beyond choke:

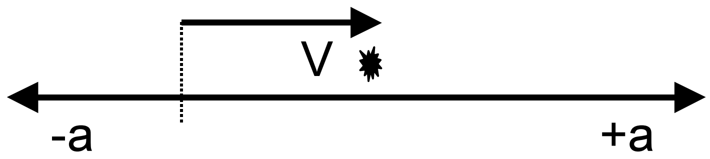
$M_{19} = 1.0$ and $p_{19} > p_a$

Same maximum mass flow

The speed of sound, $a = \sqrt{\gamma RT}$, is the speed at which small amplitude pressure waves (sound-waves) propagate through a gas in both directions (ie at $\pm a$).



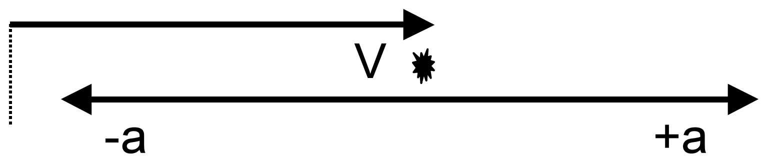
However, if the gas is moving with velocity V then:



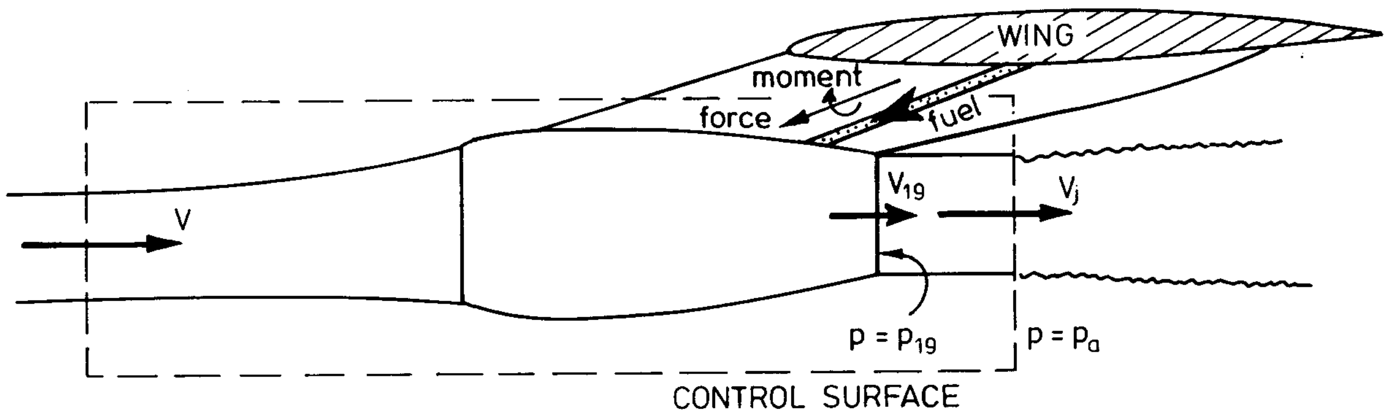
$V - a$ upstream provided $V < a$ (ie subsonic flow)

$V + a$ always downstream

"Impossible" for small waves to move upstream in a supersonic flow ($V > a$).



In flight the exit nozzle is choked (Fig 8.1):



In flight, the forward speed causes the inlet stagnation pressure to be 1.60 times the ambient static pressure. Combined with the engine pressure ratio, this ensures that the final exit nozzle is choked over the majority of flight conditions.

As the exit nozzle is choked, no "information" can pass upstream into the engine. The aeroengine performance is independent of the static pressure at the nozzle exit.

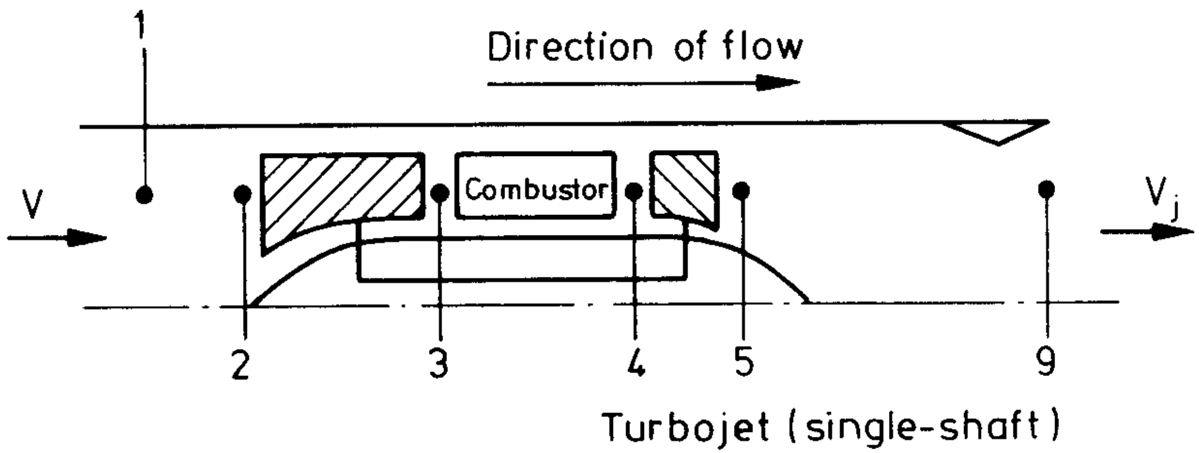
The engine is, however, affected by the inlet stagnation conditions T_{02} and p_{02}.

Calculation for a pure turbojet:

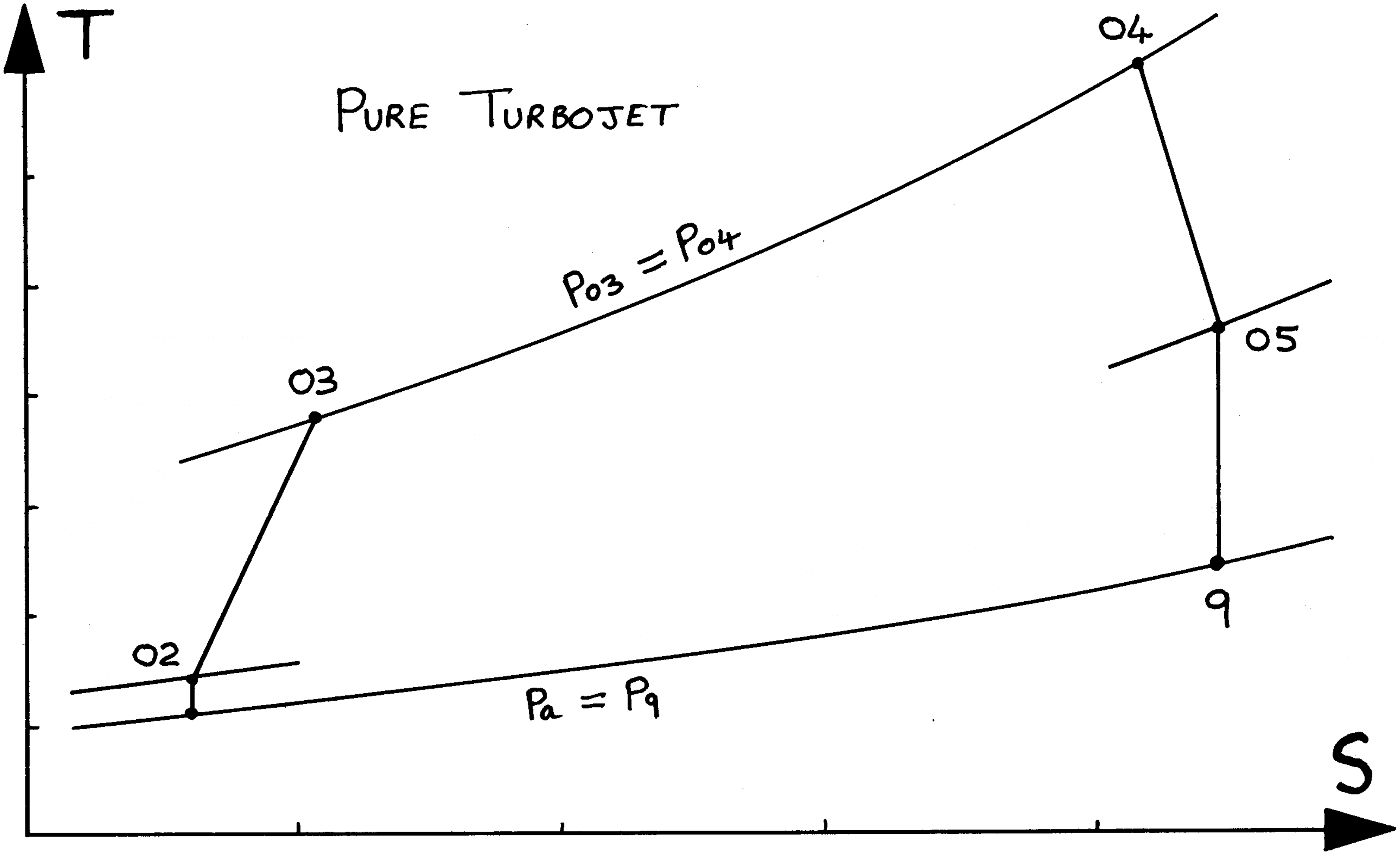
In order to have allowed the earlier studies to be undertaken with minimal complexity, various “assumptions” and “approximations” were made.

Some of these simplifications are no longer necessary, so the pure turbojet calculation (essentially exercise 4.4) will now be re-done with minimal approximations.

Numbering system through the gas turbine cycle (page 70).



TS diagram for a pure turbojet engine.



Pure turbojet flying at 31000 ft with Mach = 0.85

Standard atmosphere (31000 ft):

T_a = 227 K

p_a = 28.7 kPa

Flying at Mach = 0.85 yields:

$$V = 0.85 \sqrt{\gamma R T} = 256 \text{ m/s}$$

The stagnation conditions at engine inlet (location 02):

$$T_{02} = T_a \left(1 + \frac{\gamma - 1}{2} M^2\right) = 259 \text{ K}$$

$$p_{02} = p_a \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma - 1)} = 28.7 \times 1.60 = 45.8 \text{ kPa}$$

Pure turbojet with a compressor pressure ratio of 40 and 90% efficient:

Compressor outlet conditions (location 03):

$$p_{03} = p_{02} \times 40 = 1832 \text{ kPa}$$

$$T_{03isen} = T_{02} \times \left(\frac{p_{03}}{p_{02}}\right)^{(\gamma-1)/\gamma} = 743.1 \text{ K}$$

$$T_{03} = T_{02} + (T_{03isen} - T_{02})/\eta_{comp} = 796.8 \text{ K}$$

Compressor work input:

$$\dot{W}_c/\dot{m}_a = C_p(T_{03} - T_{02}) = 540.49 \text{ kJ/kg}$$

Pure turbojet with turbine inlet temperature 1450 K ($T_{04}/T_{02} = 5.6$, start of cruise):

Heat input required:

$$\dot{m}_f LCV/\dot{m}_a = C_p(T_{04} - T_{03}) = 656.47 \text{ kJ/kg}$$

Turbine inlet conditions (location 04):

$$p_{04} = p_{03} = 1832 \text{ kPa}$$

Shaft work energy balance:

turbine work = compressor work

$$C_p(T_{04} - T_{05}) = \dot{W}_t/\dot{m}_a = \dot{W}_c/\dot{m}_a$$

$$T_{05} = 912.7 \text{ K}$$

Calculation of turbine pressure ratio:

$$T_{05isen} = T_{04} - (T_{04} - T_{05})/\eta_{turb} = 853.0 \text{ K}$$

$$p_{05} = p_{04} \times \left(\frac{T_{05isen}}{T_{04}}\right)^{\gamma/(\gamma-1)} = 286.1 \text{ kPa}$$

Calculation of jet velocity (between locations 05 and 9):

$$p_9 = p_a = 28.7 \text{ kPa}$$

$$T_9 = T_{05} \times \left(\frac{p_9}{p_{05}}\right)^{(\gamma-1)/\gamma} = 473.2 \text{ K}$$

$$\tfrac{1}{2}V_j^2 = C_p(T_{05} - T_9)$$

$V_j = 939.9$ m/s (very fast!)

Pure turbojet propulsive and overall efficiency:

Propulsive efficiency:

$$\eta_p = \frac{\text{power to aircraft}}{\Delta KE} = \frac{2V}{V_j + V} = 0.428 \qquad \text{(very low - need bypass)}$$

(Answer from exercise 4.4: $\eta_p = 0.471$)

Overall efficiency:

$$\eta_o = \frac{\text{Thrust} \times \text{speed}}{\dot{m}_f \, LCV} = \frac{\dot{m}_a (V_j - V) \times V}{\dot{m}_f \, LCV} = \frac{(939.9 - 256) \times 256}{656.47 \times 10^3} = 0.267$$

(Answer from exercise 4.4: $\eta_o = 0.225$)